to the choice of starting values. The best approach seems to be to run the program a few times using different starting values and to select the most favorable final values.

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Studies in Chemical Process Design and Synthesis

Part VII: Systematic Synthesis of Multipass Heat Exchanger Networks

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An important process design problem is the systematic synthesis of energy-optimum and minimum-cost networks of exchangers, heaters and/or coolers to transfer the excess energy from a set of hot streams to streams that require heating (cold streams). The specific problem statements, common simplifying assumptions, and systematic synthesis techniques along with a list of stream specifications and design data for example problems reported in the literature can be found in Part III (Nishida et al., 1977). Essentially all of the work reported thus far has been limited to the use of single-pass countercurrent shell-and-tube exchangers, heaters and coolers. In this paper, a simple approach to the systematic synthesis of energy-optimum and minimum-cost networks, which may utilize multipass exchangers, heaters and coolers, is proposed and demonstrated.

THERMAL DESIGN OF MINIMUM-COST MULTIPASS EXCHANGERS, HEATERS AND COOLERS

Multipass exchangers, heaters and coolers are often used in the process industries. Such equipment allows for a great deal of flexibility to a given heat-exchange process design. For example, multiple-shell passes may be used: (i) to improve the temperature difference between hot and cold streams for a given exchanger, in which the stream flows are not parallel or countercurrent; and (ii) to decrease the amount of floor space required for a given exchanger. Multiple-tube passes may be used: (i) to increase the fluid velocity in the tubes, thereby increasing the overall heat transfer coefficient; and (ii) to increase or decrease the available heat transfer area without increasing or decreasing the shell length. The numbers of shell and tube passes are limited by the maximum allowable pressure drop and by space considerations. Further, the number of tube passes is also limited by the amount of fluid passing through the tubes and by the maximum permissible shell diameter.

The basic equation for the thermal design (i.e., finding the heat transfer area) of multipass exchangers, heaters and coolers is (Bell, 1984):

$$A = \frac{Q}{U(\text{MTD})} = \frac{Q}{UF_N(\text{LMTD})} \tag{1}$$

Here, F_N is a correction factor that is so determined that when it is multiplied by the logarithmic mean temperature difference (LMTD) for a single-pass counter-current exchanger, the product F_N(LMTD) represents the true mean temperature difference

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(MTD) of an equivalent multipass exchanger. F_N is commonly correlated graphically as a function of the number of shells N to be used in series, the number of tube passes, the capacity ratio R and thermal efficiency P. The latter are defined by:

$$R = \text{capacity ratio} = \frac{W_c}{W_h} = \frac{T_h - T_h^*}{T_c^* - T_c}$$

$$P = \text{thermal efficiency} = \frac{T_c^* - T_c}{T_h - T_c} \quad (2)$$

For a thermodynamically feasible and efficient multipass exchanger (heater or cooler), N is typically chosen such that F_N is at least 0.8 at the given values of R and P (see the discussion below for an important exception to this heuristic). Also, the use of up to six shells in series is quite common in the process industries, particularly in crude-unit preheat recovery trains.

Minimum-Cost Multipass Exchangers (Heaters or Coolers)

For estimating the heat transfer area and the corresponding investment cost of a multipass exchanger with N shells in series, it is not important to know the exact number of tube passes as long as the latter is at least twice the number of shells. This is evident by observing the available graphical correlations of F_N as functions of R and P such as those found in Bell (1984), which show only a negligible difference among the values of F_N at different tube passes for the same values of N, R and P. Next, to find the number of shells N which corresponds to a minimum-cost multipass exchanger, it is important to recognize two points. First, when replacing a single-pass countercurrent exchanger of a heat transfer area A (m^2) and an investment cost aA^b($) with a multipass exchanger of N shells in series and a LMTD correction factor F_N, the heat transfer area and the corresponding investment cost of each of the shells are commonly assumed to be identical, being equal to (A/F_NN) m^2 and $a(A/F_NN)^b$\$, respectively. The total investment cost for N shells in series is then $N \cdot a(A/F_NN)^b$\$. Secondly, for thermodynamically feasible designs of multipass exchangers with $0.8 \le F_N \le 1.0$ and $N = 2$ to 6, it can be shown that:

$$aA^b < N \cdot a\left(\frac{A}{F_N N}\right)^b < (N+1) \cdot a\left[\frac{A}{F_{N+1}(N+1)}\right]^b \quad (3)$$

for $0 < b < 1$ (b is typically 0.6). This inequality suggests that the investment cost of a multipass exchange of $(N + 1)$ shells is always higher than that of N shells, and the latter is always higher than that of a single-pass countercurrent exchanger designed at the same values of R and P. Consequently, a key step in the thermal design of minimum-cost multipass exchangers is to find the required number of shells in series.

Estimating the Number of Shells, *N*

Bell (1978) has proposed a graphical method based on the operating lines in stagewise process design, to estimate the value of N. This procedure utilizes the inlet and outlet temperatures of both hot and cold streams as illustrated in Figure 1, in which N is about 3. In this work, it has been found that for $N > 3$, Bell's method frequently cannot be used to predict feasible designs of multipass exchangers. Specifically, by following the procedure used in the development of the Kremser equation in stagewise process design (McCabe and Smith, 1976), it can be shown that the operating line in Figure 1 can be represented analytically by:

$$\frac{RP}{1 - RP} = \frac{T_h - T_h^*}{T_h^* - T_c} = R + R^2 + \ldots + R^N = \frac{R(1 - R^N)}{1 - R} \quad (4)$$

Simplifying Eq. 4 gives an explicit expression for finding N from given values of R and P by Bell's method:

$$N = \frac{ln\left(\frac{1 - R}{1 - RP}\right)}{ln\, R} \quad (5)$$

and the relationship

$$P = \frac{1 - R^N}{1 - R^{N+1}} \quad (6)$$

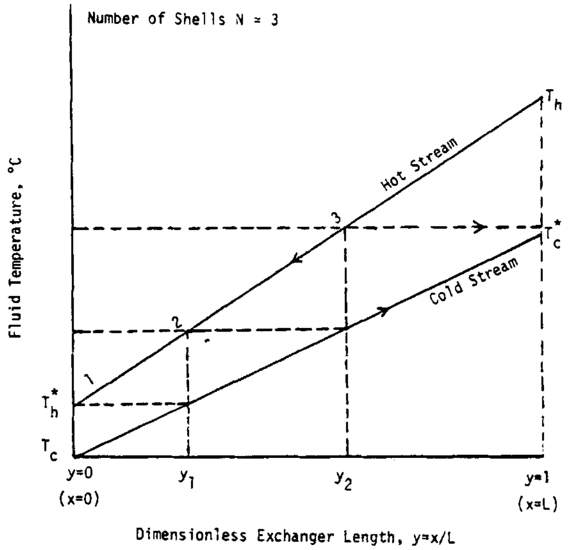

Figure 1. Bell's method (1978) of finding the number of shells in a multipass exchanger (heater or cooler).

To evaluate the applicability of Bell's method, P is calculated from Eq. 6 for given values of R and N, and the values of F_N corresponding to a wide range of values of P, R and N are then read from available graphical correlations (Bell, 1984). The results show that for $N = 4$ to 6, $R \ge 2.0$ or $P \le 0.50$, Bell's method results in $F_N < 0.8$ and may fail to predict feasible designs of multipass exchangers.

Based on the preceding discussion and the inequality given by Eq. 3, the following procedure is recommended for estimating the number of shells in series. First, one assumes a value of N (≤ 6) and determines the value of F_N from the given values of R and P by using available correlations (Bell, 1984). If the resulting $F_N < 0.8$, one then increases the value of N by increments of one until F_N is at least 0.8. For the value of N which corresponds to a transition from $F_N < 0.8$ to $F_N \ge 0.8$, this value of N is the number of shells in a minimum-cost multipass exchanger.

In applying this procedure, it is important to check the graphical correlations of $F_N = f(R, P, N)$ to ensure that F_N is not very sensitive to small changes in values of R and P near the given values of R and P at the selected value of N. In other words, F_N should always be checked to see that it does not fall into an asymptotic–F_N region in the graphical correlation of $F_N = f(R, P, N)$ at given values of R and P (Taborek, 1979); one should consider the lower limit of $F_N = 0.8$ as an *apparent, but not absolute*, minimum value of F_N. If F_N is very sensitive to small changes in values of R and P, the chosen value of N may not predict a feasible design of multipass exchanger even when $F_N > 0.8$. In the latter case, one should further increase the value of N by increments of one and check again if F_N is still very sensitive to small changes in R and P at the selected value of N. When F_N is very sensitive to small changes in values of R and P for $N = 2$ to 6, or when $F_N < 0.8$ at given values of R and P for $N = 2$ to 6, the use of a single-pass countercurrent exchanger (heater or cooler) is recommended.

CHARACTERISTICS OF STREAM MATCHING IN A MINIMUM AREA OR NEARLY MINIMUM COST MULTIPASS HEAT EXCHANGER NETWORK

In Appendix, it is shown by an example of a network of exchangers, heaters and/or coolers of one shell pass and two tube passes (i.e., 1–2 type) that the sequence of stream matching in a minimum area or nearly minimum cost multipass network follows the thermodynamic matching rule of Part III: "Hot process and utility streams, and cold process and utility streams are to be matched consecutively in a decreasing order of their stream temperatures."

TABLE 1. DESIGN AND COST: MULTIPASS VERSIONS OF THE OPTIMUM AND SUBOPTIMUM SINGLE-PASS NETWORKS FOR THE 4SP1 PROBLEM

Network	Service	No. of Shells, N	LMTD Correction Factor, F_N	Area (m^2)	Investment Cost ($)
No. 1	E_1	3	0.874	23.06	16,103
	E_2	2	0.924	8.38	7,215
	E_3	2	0.925	10.06	8,045
	E_4	2	0.993	4.32	4,643
	E_5	2	0.980	6.64	6,057
	C_1	2	0.976	6.29	6,071
	H_1	1	—	3.72	3,203
		(Total Network Cost = $15,560/yr)			$51,337
No. 2	E_1	3	0.874	23.06	16,103
	E_2	2	0.860	10.07	8,410
	E_3	2	0.887	12.92	9,587
	E_4	2	0.980	6.64	6,057
	C_1	2	0.976	6.29	6,071
	H_1	1	—	3.72	3,203
		(Total Network Cost = $15,370/yr)			$49,431
Final	E_1	4	0.888	29.28	20,652
	E_2	2	0.828	19.88	12,939
	E_3	2	0.980	6.64	6,056
	C_1	2	0.976	6.29	6,071
	H_1	1	—	3.72	3,203
		(Total Network Cost = $15,317/yr)			$48,921

SYSTEMATIC SYNTHESIS OF ENERGY-OPTIMUM AND MINIMUM-COST MULTIPASS HEAT EXCHANGER NETWORKS

A simple approach to the systematic synthesis of multipass heat exchanger networks can now be proposed. First, the given synthesis problem is solved initially as one which employs only single-pass services (exchangers, heaters and coolers) by using the thermoeconomic approach of Part VI (Pehler and Liu, 1984). The necessary heat transfer areas of different single-pass services in the resulting energy-optimum and minimum-cost network are determined. Secondly, with the resulting heat transfer areas of single-pass services, the minimum-cost multipass services, which achieve the same design objectives (with the same values of R and P), are found by finding the required number of shells N and the corresponding value of F_N. The corresponding investment costs of different multipass services are then determined from $N{\cdot}a(A/F_N N)^b$. Note that in applying the thermoeconomic approach of Part VI, *minimum* amounts of heating and cooling utilities are to be used in a given synthesis problem; and the corresponding utility operating cost is minimum, which can be easily found prior to synthesizing the network.

ILLUSTRATIVE EXAMPLES AND CONCLUSIONS

Figure 2 illustrates the minimum cost (final) and two nearly minimum cost single-pass networks for a four-stream problem, 4SP1 (Lee et al., 1970), synthesized by the thermoeconomic approach of Part VI. These networks are energy-optimum and have an identical, minimum utility operating cost of $10,426 per year. By using the proposed approach, the results of synthesizing the corresponding multipass networks with an apparent minimum F_N of 0.8 are summarized in Table 1. The utility operating costs of all multipass networks are identical and equal to the minimum utility cost of the single-pass networks. The total annual costs of these energy-optimum multipass networks differ only by a few percent; this small cost difference is similar to that observed for the single-pass networks shown in Figure 2.

Based on the above comparison and the results obtained for other reported synthesis problems with five to ten process streams, including problem 5SP1, 6SP1 and 7SP1 (Masso and Rudd, 1969), and problem 10SP1 (Pho and Lapidus, 1973), a number of general conclusions and observations can be summarized as follows.

1) For all problems solved, F_N for each exchanger, heater or cooler was not sensitive to small changes in values of R and P for N = 2 to 6 under the given problem specifications and the use of an apparent minimum F_N of 0.8 appeared to be feasible.

2) The relative cost ranking of different energy-optimum single-pass networks synthesized by the thermoeconomic approach of Part VI has been found to generally preserve in the corresponding multipass networks synthesized by the proposed method

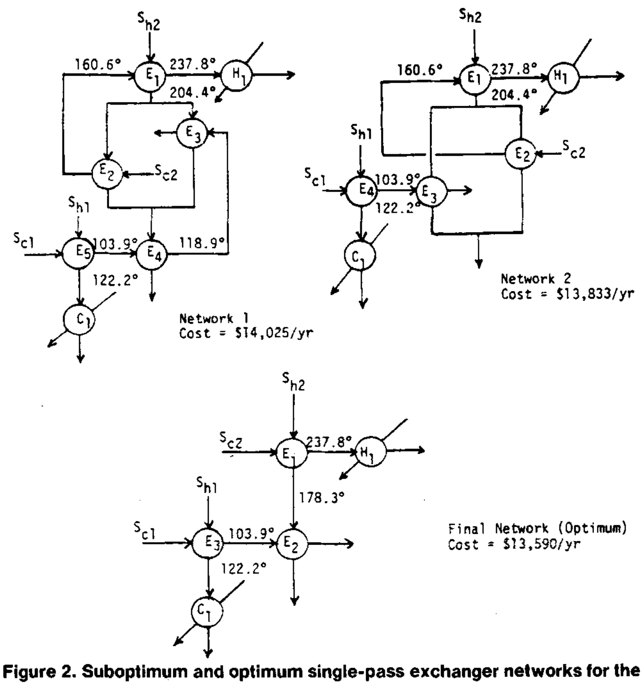

Figure 2. Suboptimum and optimum single-pass exchanger networks for the 4SP1 problem.

when an apparent minimum F_N of 0.8 was used. Slightly different cost ranking of the resulting multipass networks has been noted for some cases when different apparent minimum values of F_N were used.

3) For all problems solved, the total annual costs of both single-pass and multipass energy-optimum networks differ only by a few percent. The multipass networks synthesized by the proposed method can be readily used for further multiobjective design optimization to select the best network for a specific application, based on additional performance criteria other than the total cost such as the ease of startup and shutdown, and the sensitivity to varying stream inlet temperature and flow rate, etc.

The preceding conclusions and observations have been confirmed by results from computer implementation of the proposed synthesis method on HEXTRAN, a commercial simulation program for heat exchanger networks described by Challand et al. (1981).

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APPENDIX

Characteristics of Stream Matching in Energy-Optimum, Minimum Area or Nearly Minimum-Cost Multipass Heat Exchanger Networks

The characteristics of stream matching in energy-optimum, minimum-area or nearly minimum-cost multipass heat exchanger networks can be illustrated by considering, for example, a network of 1-2 type shell-and-tube exchangers, heaters and/or coolers (one shell pass and two tube passes). In particular, the heat transfer area of 1-2 type shell-and-tube exchangers (heaters or coolers) can be found from the analytical expression (Bowman et al., 1940):

$$A = \frac{W_c}{U\sqrt{1+R^2}} \log \frac{2 - P(R + 1 - \sqrt{1+R^2})}{2 - P(R + 1 + \sqrt{1+R^2})} \quad \text{(A1)}$$

where R and P are defined by Eq. 3 and

$$Q = W_h(T_h - T_h^*) = W_c(T_c^* - T_c) \quad \text{(A2)}$$

Combining Eqs. A1 and A2 gives

$$A = \frac{W_c}{U\sqrt{1+R^2}} \log \frac{T_h - T_c - \dfrac{\beta}{2}(R + 1 - \sqrt{1+R^2})}{T_h - T_c - \dfrac{\beta}{2}(R + 1 + \sqrt{1+R^2})} \quad \text{(A3)}$$

where $\beta = Q/W_c$. In what follows, the procedure presented in Nishida et al. (1971) is adapted to show that in a minimum-area or nearly minimum-cost network of 1-2 type shell-and-tube services, the stream matching follows the guideline of the thermodynamic matching rule of Part VI (Pehler and Liu, 1984): "Hot process and utility streams, and cold process and utility streams are to be matched consecutively in a decreasing order of their stream temperatures."

Consider a permutation of hot and cold process and utility streams between two services (exchangers, heaters or coolers) involved in the minimum-area or nearly minimum-cost network, say, from (S_{hi},S_{ci}) and (S_{hj},S_{cj}) to (S_{hi},S_{cj}) and (S_{hj},S_{ci}), where the notation (S_{ha},S_{cb}) denotes an exchange between hot stream S_{ha} and cold stream S_{cb}. Since the other part of the network remains the same as before, a change in the heat transfer area caused by the permutation can be written from Eq. A3 as follows:

$$\Delta A = \frac{W_c}{U\sqrt{1+R^2}} \left[\left(\log \frac{T_{hi} - T_{cj} - a}{T_{hi} - T_{cj} - b} + \log \frac{T_{hj} - T_{ci} - a}{T_{hj} - T_{ci} - b} \right) - \left(\log \frac{T_{hi} - T_{ci} - a}{T_{hi} - T_{ci} - b} + \log \frac{T_{hj} - T_{ci} - a}{T_{hj} - T_{cj} - b} \right) \right] \quad \text{(A4)}$$

where $C = \beta(R + 1 - \sqrt{1+R^2})/2$ and $d = \beta(R + 1 + \sqrt{1+R^2})/2$.

Rearranging Eq. A4 gives

$$\Delta A = \frac{W_c}{U\sqrt{1+R^2}} \log \times \frac{(T_{hi} - T_{cj} - c)(T_{hj} - T_{ci} - c)(T_{hi} - T_{ci} - d)(T_{hj} - T_{cj} - d)}{(T_{hi} - T_{cj} - d)(T_{hj} - T_{ci} - d)(T_{hi} - T_{ci} - c)(T_{hj} - T_{cj} - c)} \quad \text{(A5)}$$

If the resulting network is still a minimum-area or nearly minimum-cost one, than $\Delta A \geq 0$. From Eq. A5, $\Delta A \geq 0$ implies that:

$$(d - c)[(T_{hi} - T_{ci} - d) + (T_{hj} - T_{cj} - c)](T_{hi} - T_{hj})((T_{ci} - T_{cj}) \geq 0 \quad \text{(A6)}$$

Note that $(d - c) = \beta\sqrt{1+R^2} > 0$. Further, since it is assumed that heat is transferred from hot to cold streams, one can write

$$T_{hi} - T_{ci} - d > 0 \qquad T_{hj} - T_{cj} - c > 0$$

Therefore, Eq. A6 can be simplified to give

$$(T_{hi} - T_{hj})(T_{ci} - T_{cj}) \geq 0$$

which implies that $T_{hi} \geq T_{hj}$ if $T_{ci} > T_{cj}$. By repeating the same analysis for all possible pairs of two-stream exchange, it is possible to show that $\Delta A \geq 0$ implies

$$T_{hi} > T_{h2} > \ldots\ldots > T_{hm}$$
$$T_{c1} > T_{c2} > \ldots\ldots > T_{cm} \quad \text{(A7)}$$

Therefore, the minimum-area or nearly minimum-cost network may be represented by the following pairs of exchangers which satisfy Eq. A7: (S_{h1},S_{c1}), (S_{h2},S_{c2}) ..... (S_{hm},S_{cm}). This sequence of stream matching exactly follows the guideline of the thermodynamic matching rule.

NOTATION

a,b	= constants in the network investment cost function
A	= the heat transfer area of a single-pass countercurrent exchanger, heater or cooler, m^2
A_N	= the heat transfer area for each shell-pass of a multipass exchanger, heater or cooler with N shells in series, m^2
F_N	= the LMTD correction factor for a multipass exchanger, heater or cooler of N shells, dimensionless
L	= the length of an exchanger, a heater or a cooler, Figure 1, m
N	= number of shells in a multipass exchanger, heater or cooler
P	= thermal efficiency of an exchanger, a heater or a cooler, Eq. 2, dimensionless
Q	= the rate of heat transfer between hot and cold streams in an exchanger, a heater or a cooler, Eq. 1, kW
R	= capacity ratio defined as the ratio of the heat capacity flow rate of hot stream to that of cold stream, Eq. 2, dimensionless
T	= temperature, °C
T_{hi},T_{hi}^*	= the inlet and outlet temperatures of the ith hot stream, respectively, °C
T_{cj},T_{cj}^*	= the inlet and outlet temperatures of the jth cold stream, respectively, °C
U	= the overall heat transfer coefficient for an exchanger, a heater or a cooler, Eq. 1, $W/m^2\cdot°C$
W_{hi},W_{cj}	= the heat capacity flow rate (heat capacity multiplied by mass flow rate) of the ith hot and jth cold process streams, respectively, kW/°C
x	= the distance along the length of an exchanger, Figure 1, m

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A Correction Factor for Mass Transfer Coefficients for Transport to Partially Impenetrable or Nonadsorbing Surfaces

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INTRODUCTION

Most correlations for liquid-solid mass transfer coefficients in packed or fluidized beds or for surfaces having other various geometries are based primarily upon measurement of dissolution rates using soluble packings (Evans and Gerald, 1953; Goto and Smith, 1975; Goto et al., 1975; Hirose et al., 1976; Kataoka et al., 1972; Lemay et al., 1975; Specchia et al., 1976, 1978; Sylvester and Pitayagusarn, 1975; Van Krevelen and Krekels, 1948; Wilson and Geankopolis, 1966). Other techniques that have been used to a lesser extent include those based upon electrochemical methods (Colquhoun-Lee and Stepanek, 1978; Karabelas et al., 1971), tracer methods (Tan and Smith, 1982), or adsorption using a chemical reaction in the mass transfer controlled regime (Snider and Perona, 1974). Critical reviews and a summary of the significant findings are available (Charpentier, 1976, 1981; Karabelas et al., 1971; Dwivedi and Upadhyay, 1977; Wakao and Funazkri, 1978). Care is usually taken to ensure that the solid surfaces used are smooth on the scale of the boundary layer thickness so that the actual interfacial area available for transport corresponds to the geometric area on which the calculated mass transfer coefficients are based. These mass transfer coefficients obtained are essentially for nonporous solid surfaces and are often used to predict mass fluxes in systems where only a fraction of the external area of the solid participates in the transport. Such is the case of a discretely dispersed catalyst on a nonadsorbing, nonporous support or the case of porous catalysts. Common engineering practice is to base the mass transfer coefficients on the total external area. The question then arises whether the mass transfer coefficient obtained in systems where total external area participates in transport, k^o_{mt}, should be modified for the case where the transport occurs only to a fraction f of the external area but the coefficient k_{mt} is still based on the total area. Conventional wisdom (Herskowitz et al., 1979) suggests that the two are related by:

$$k_{mt} = f\, k^o_{mt} \tag{1}$$

Equation 1 implies that if the mass transfer coefficients are based per unit active area available for transport, they would be unchanged irrespective of whether the whole surface or only part of it is active in the transport process. This would suggest that the following equality be satisfied:

$$k^o_{ma} = k^o_{mt} = k_{ma} = \frac{k_{mt}}{f} \tag{2}$$

where k_{ma} is the mass transfer coefficient for a partially active surface based per unit area of active surface. Our objective is to test this hypothesis based on a simple two-dimensional model for mass transfer to a solid surface.

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MODEL

We consider here the simplest two-dimensional situation of transport by diffusion through a boundary layer of constant thickness δ to a nonuniform surface as shown in Figure 1. The surface is nonuniform in the sense that it has periodically distributed active and inactive areas. The half-width of the active area is denoted here by p and the half-width of the inactive area by $(w - p)$. By placing the origin of the coordinate system at the center of an active area, the lines of symmetry are located at $x = 0$ and $x = w$.

The active areas may represent pore mouths or active catalyst crystallites. We assume that the bulk fluid at $y = \delta$ is maintained at a constant concentration of the diffusing species $c = c_b$. The active area of the surface at $y = 0$ for $0 \leq x < p$ exists at some finite

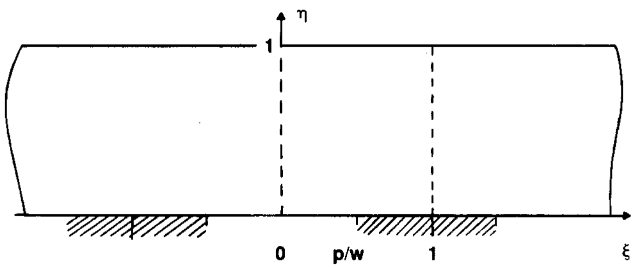

Figure 1. Diffusion to a partially impenetrable surface.